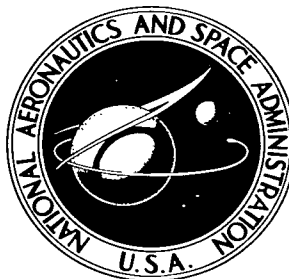


NASA TECHNICAL NOTE



NASA TN D-3777

NASA TN D-3777

LOAN COPY: RETURN TO
AFWL (WLIL-2)
KIRTLAND AFB, N MEX

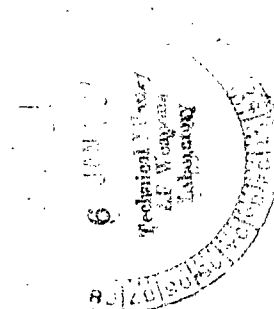
0130544



TECH LIBRARY KAFB, NM

STRUCTURAL OPTIMIZATION WITH PROBABILITY OF FAILURE CONSTRAINT

by Dhanjoo N. Ghista
Ames Research Center
Moffett Field, Calif.





0130544

NASA TN D-3777

STRUCTURAL OPTIMIZATION WITH PROBABILITY
OF FAILURE CONSTRAINT

By Dhanjoo N. Ghista

Ames Research Center
Moffett Field, Calif.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 -- Price \$1.00

STRUCTURAL OPTIMIZATION WITH PROBABILITY OF FAILURE CONSTRAINT

By Dhanjoo N. Ghista
Ames Research Center

SUMMARY

Minimum weight structural design consists in determining the set of design parameters of the structure which makes the weight of the structure a minimum without violating the constraints imposed on the structure.

In a structural optimization problem, where the constraints are the limiting stresses, failure is absolutely prevented, that is, with a probability of 1. Such absolute safety not only limits the minimization of weight but is idealistic and therefore impractical, for in actual systems limits must be imposed within which failure can occur. The limiting stress constraint, having a failure probability of 0, is hence replaced by a limiting probability of failure constraint.

In this paper, the concept of probability of failure is explained for a general structural system under simultaneous and alternative load systems. It is then shown how the probability of failure constraint can be formulated in terms of the structural design parameters. This constraint is developed in detail for optimizing a simple truss whose design parameters are the section areas of the members. The optimizing weight function and the constraint in terms of the design parameters define the mathematical model of the structural optimization problem. The structural synthesis method is suggested for solving the mathematical programming problem, and it is shown how the probability of failure constraint can be incorporated in the synthesis technique. The application of the constraint is illustrated by means of two simple examples with reference to the truss.

INTRODUCTION

Minimum weight structural design consists in determining the set of design parameters of the structure which makes the weight of the structure a minimum and at the same time ensures that the structure is safe under the governing load system. To determine whether a structure is safe or not, the computed design stresses are compared with the permissible stresses. The structure will fail if the stress in any component exceeds the allowable limit.

In a structural optimization problem, where the constraints are the limiting stresses, the extent to which the weight can be diminished is governed, then, by the limiting stress constraint. Imposing this constraint prevents failure absolutely, that is, with a probability of 1. Now the design

stresses in the components of a structure are a function of the loading, the structural design parameters, namely, the sizes of the elements and the material properties. The permissible stress or the failure stress for a particular material is a physical property of the material. The properties have to be estimated to represent all possible relevant conditions. Past experience shows that it is possible to represent a material property by a frequency distribution. Such a distribution, then, furnishes its most probable value, that is, its mean and its probable range of fluctuation represented by its standard deviation. Similarly, the applied loading is a random variable and may also be represented by a frequency distribution. This enables an assessment to be made of the most probable conditions, fluctuation ranges, and the probability that certain extremes will occur.

It is seen, then, that to prevent failure absolutely, extreme fluctuations of load and permissible stresses would have to be taken into account, that is, the upper and lower extremes of the load and permissible stress. Such absolute safety not only limits the minimization of weight but is idealistic. On actual systems, limits must be imposed within which failure can occur. For the most conservative system, failure can be made highly improbable by making its probability small but it is impractical to attempt to prevent it absolutely. The limiting stress constraint, having a failure probability of 0, is hence replaced by a limiting probability of failure constraint.

In assigning probability of failure to a structure, a compromise has to be made between the level of economy to be maintained and the amount of risk a designer is prepared to undertake. Consider now, as examples, two structural systems: (1) a system of limited purpose mission with a large number of units per mission, such as missiles, (2) a multipurpose single trial system, such as a Mars probe lander. In the case of missiles, say if 50 missiles were to be deposited over an area, it does not matter if 1 or 2 do not come off and fail to accomplish the mission. So it pays not to overdesign the missile and to allow a possibility of failure of 1 or 2 out of 50 as a result of stresses exceeding their limiting values. This means that in the design of the missile, the probability of failure should be less than or equal to 0.02 or 0.04. Such a constraint, as opposed to a failure probability of 0, results in considerable saving in the weight of the missile. In the case of a single trial system such as a Mars probe lander, the probability of failure would have to be extremely small, say 1 in 10^7 .

A reliability approach to structural design has been recognized increasingly during the past decade (see Bibliography); the researchers have stressed the problems of introducing realistic safety factors and of proportioning a structure to meet a prescribed reliability. This literature indicates the need for a method of systematic determination of the minimum weight design of a structure within limits of a prescribed probability of failure. This paper, then, endeavors to present a compact mathematical model of the structural optimization problem wherein the merit function to be optimized is the structural weight, and the probability of failure of the structure is expressed in terms of the design parameters and employed as a constraint in the optimization problem.

In this paper, the concept of probability of failure is explained for a general structural system under simultaneous and alternative load systems. It is then shown how the probability of failure constraint can be formulated in terms of the structural design parameters. This constraint is developed in detail for the optimization of a simple truss whose design parameters are the section areas of the members. The mathematical model of the structural optimization problem is defined by the optimizing weight function and the constraint in terms of the design parameters. The structural synthesis method is suggested for solving the mathematical programming problem, and it is shown how the probability of failure constraint can be incorporated in the synthesis technique. The application of the constraint is illustrated by means of two simple examples with reference to the truss.

NOTATION

A_i	i th design parameter (area of the i th member of the truss, with $i = 1, 2$ for bars OA and OB, respectively)
E	modulus of elasticity
F_i	failure stress of the i th element, taken to be a normally distributed random variable
P_{ij}	probability of failure of the i th element under j th loading
P_{FQ_j}	probability of failure of the structure under loading Q_j
P_F	probability of failure of the structure
$Q_j (j=1,2)$	j th loading, taken to be a normally distributed random variable
\bar{Q}_j	mean of random variable Q_j
R_{ij}	force in i th member under j th loading
W	structural weight
f_i	applied stress in i th element
g_{ij}	random variable $[F - (R_{ij}A_i)]$
l	length of either member of the truss
ρ	density of material for the truss
σ_{Q_j}	standard deviation of random variable Q_j

ANALYSIS

Factors Influencing the Probability of Failure of the Structure

Dead load. - Factors causing variation in dead load are the dimensions of the structure and the specific weight of the materials.

Live load. - To have a complete representation of live load, it is necessary to know the most probable conditions of loading with a high frequency of occurrence and, at the same time, to allow for certain adverse conditions that could occur. For example, if the loading represents wind load, one must know what percentage of time the wind velocity is in a certain range and also the magnitude of, say, hundred-year gales.

The variation in loading, its most probable value, and estimates of certain extreme cases can be represented by a frequency distribution with given mean and standard deviation.

Structural stiffness. - The structural stiffness depends upon the structural configuration and the elasticity of the members. The modulus of elasticity of the structure is a constant, statistically only. It can be represented by a frequency distribution with given mean and standard deviation. The sizes and the dimensions of the structures can have definite variations about their mean values if their tolerances are specified. The design stress in a member is then a function of the loading and the structural stiffness.

Failure stress. - The failure stress is a physical property of the material. Its variability depends on the control exercised during manufacturing process. It, too, is represented by a frequency distribution.

Development of Probability of Failure Constraint

Consider a structure made up of r elements or components. Let Q_1 and Q_2 represent two independent loading systems; Q_1 and Q_2 are random variables, normally distributed with means \bar{Q}_1 and \bar{Q}_2 and standard deviations σ_{Q_1} and σ_{Q_2} , respectively. It could happen that both Q_1 and Q_2 would act simultaneously as dead and live loads on the structure. On the other hand, the structure could be undergoing alternative loadings, in which case either Q_1 or Q_2 would be acting but not together. Assume that the mode in which each component or element fails is known. The corresponding failure stress (of the i th element) is denoted by F_i , which is taken to be normally distributed with mean \bar{F}_i and standard deviation σ_{F_i} . It is required that the probability of failure of the structure not exceed a fixed value α .

Let the parameter A_i represent the size of a component to be determined. The design parameters A_i constitute the variables for the optimization problem. For a given structure, the design stresses are functions of member sizes A_i and the elasticity E of the material. For the sake of

convenience, A_i and E are not taken to be random variables, but constants, in order to simplify the determination of the frequency distribution of the applied stress. The weight (W) of the structure is expressed as a function of the design parameters A_i . The optimization problem consists in determining A_i so that W is a minimum and the probability of failure of the structure P_F is less than or equal to α .

Let P_{FQ1} and P_{FQ2} represent the probabilities of failure of the structure under loadings $Q1$ and $Q2$, respectively. If the loadings $Q1$ and $Q2$ are such that they may or may not act simultaneously, then the structure can fail when either $Q1$ or $Q2$ or both $Q1$ and $Q2$ are acting on the structure. Then, the desired probability of failure of the structure is the probability that it will fail under $Q1$ or $Q2$ or $Q1$ and $Q2$ acting together [P_{FQ1} or $Q2$ or ($Q1$ and $Q2$)] and is given by

$$\left[P_{FQ1} \text{ or } Q2 \text{ or } (Q1 \text{ and } Q2) \right] = P_{FQ1} + P_{FQ2} - P_{FQ1}P_{FQ2} \quad (1)$$

For any loading, the probability of failure of the structure (P_F) is expressed in terms of the probabilities of failure of its constituent elements (P_i) as follows:

$$P_F = 1 - (1 - P_1)(1 - P_2) \dots (1 - P_i) \dots (1 - P_r) \quad (2)$$

A structural element will fail when the stress (f_i) in it, due to the applied loading, exceeds the allowable limit F_i . Let P_i denote the probability of failure of an element. Thus

$$P_i = P(f_i > F_i) = P(F_i - f_i < 0) \quad (3)$$

The quantity of interest is, then, the random variable ($F_i - f_i$). Let us consider the distribution of f_i . It is a function of the loading and the stiffness of the structure. The latter quantity is assumed to be a constant, for the sake of simplicity. Hence, if the loading is normally distributed, f_i will also be normally distributed. Also, F_i is normally distributed. The random variable ($F_i - f_i$) is thus normally distributed. Now given the

values of the design parameters A_i , the random variables $(F_i - f_i)$ all become determinate; in other words, the means and standard deviations are known. It is then possible to determine the probabilities (P_i) of the constituent elements and hence from equations (1) and (2) the probability of failure of the structure, that is, either P_{FQ1} and $Q2$ or P_{FQ1} or $Q2$, as the case may be. Thus, P_F is a function of the design parameters A_i .

$$P_F = \left(P_{FQ1} \text{ and } Q2 \text{ or } P_{FQ1} \text{ or } Q2 \text{ or } Q1 + Q2 \right) = \Phi(A_1, A_2, \dots, A_i, \dots, A_r) \quad (4)$$

The weight (W) of the structure can also be expressed in terms of the design parameters:

$$W = W(A_1, A_2, \dots, A_i, \dots, A_r) \quad (5)$$

The optimization problem consists in optimizing the function W (eq. (5)), subject to the constraint that P_F (eq. (4)) is less than or equal to the permissible limit α . Now that the optimizing function and the constraint have been expressed in terms of the design parameters, the stage is set for the optimizing process. To this end, the method of structural synthesis (ref. 2) can be employed.

Optimization Method

The structural optimization problem being considered is a mathematical programming problem for determining the optimum set of design parameters that minimizes the weight of the structure and restricts its probability of failure below a certain value. The method of structural synthesis is suggested for solving this problem.

The synthesis method consists in repeated and systematic redesign of a structure by means of its governing principles without violating the constraints imposed on the structure. A trial set of parameters is initially selected. For this set, the probability of failure of the structure under either $Q1$ or $Q2$ is calculated and checked to see if it exceeds the allowable limit α . If not, the assumed values of the design parameters are altered so as to diminish the weight. This modification is made according to the governing principles of the method. With every modification, the probability of failure is calculated and checked as to whether it exceeds the allowable limit. The optimum set of parameters is said to have been reached when it is no longer possible to cause modifications in them, so as to diminish the weight of the structure without violating the probability of failure criterion $P_F \leq \alpha$.

The method is qualitatively illustrated with a simple problem which requires only two design parameters to define a given design. Such a situation can be easily graphed so that the problem constraints and the

solution technique can be visualized. The design problem is to determine the optimum values of areas A_1 and A_2 of the two-bar truss (fig. 1) so as to minimize (without the probability of failure exceeding α) the weight of the truss. Figure 2 is a plot in what may be called the design parameter space. The design parameters A_1 and A_2 are the abscissa and ordinate, so that any point in the design space represents a design. The weight contours

$$W(A_1, A_2) = C_m$$

are plotted in the design space for different values (C_m) of the structural weight. The constraint curve

$$P_F(A_1, A_2) = \alpha$$

is also shown plotted in the design space. Notice that the feasible region consisting of the set of all points of designs is bounded by the inequality

$$P_F(A_1, A_2) < \alpha$$

Clearly, the design satisfying the constraint which has the minimum weight occurs at M, the point of tangency of the minimum possible valued weight contour to the constraint curve. When there are more than two design parameters, such a graphical solution is not possible. In such a case, the synthesis method provides means of systematic convergence to optimum design from a trial design. The two-dimensional case makes it possible to illustrate graphically the principles of this technique by means of figure 2.

In terms of the design space, a design point could be

- (1) A free point (e.g., points F_n , $n = 1, 2, \dots$), if it lies in the feasible region
- (2) A bound point (e.g., point B), if it lies on the constraint curve
- (3) A violation point V, if it violates the constraint.

In essence, the method consists of a combination of three directives in terms of effecting changes in the design parameters at a current point, depending on whether it is a free point, a bound point, or a violation point (see ref. 2 for a more elaborate presentation of the method). They are as follows:

- (a) If the current design point is free (F_n), changes are so made that the rate of change of weight is a maximum. This corresponds to a travel in the design space in the direction of the gradient of the weight function.
- (b) If the current design point is a bound point (B), travel to an alternate point on the same weight contour and adopt directive (a).

- (c) If the current design point is a violation (V), retreat to the previous design point and travel to an alternate point on the same weight contour from where it is possible to adopt directive (a).

Figure 2 shows a probable convergence path from a trial design point F_1 to the optimum design point M .

Optimization of a truss for a given probability of failure constraint.- Consider a statically determinate truss AOB (fig. 1). The given data consist of (1) two loadings Q_1 and Q_2 which may or may not occur simultaneously. Both Q_1 and Q_2 are random variables, normally distributed with means \bar{Q}_1 and \bar{Q}_2 and standard deviations σ_{Q_1} and σ_{Q_2} , respectively; (2) failure stress F , which is a random variable normally distributed with mean \bar{F} and standard deviation σ_F ; and (3) the probability α that the structure will fail under either Q_1 or Q_2 or (Q_1 and Q_2), namely $[P_{FQ_1 \text{ or } Q_2 \text{ or } (Q_1 \text{ and } Q_2)}]$.

The design parameters to be determined are areas A_1 and A_2 of the bars OA and OB, respectively. Areas A_1 and A_2 are unknown constants. It is required to minimize the weight (W) of the truss subject to the constraint that the probability that it fails under Q_1 or Q_2 does not exceed α .

If ρ is the density of the material and l the length of each bar, then the weight (W) of the truss is given by

$$W = \rho l(A_1 + A_2) \quad (6)$$

Let R_{11} and R_{21} denote the forces in the bars OA and OB under the load Q_1 , and R_{12} and R_{22} , the forces in OA and OB under the loading Q_2 . Let P_{11} and P_{21} denote the probability of failure of bars OA and OB, respectively, under the loading Q_1 ; and P_{12} and P_{22} denote the probabilities of failure of bars OA and OB, respectively, under the loading Q_2 . Now,

$$\begin{aligned} P_{ij} &= P\left(\frac{R_{ij}}{A_i} > F\right) \\ &= P\left(F - \frac{R_{ij}}{A_i} < 0\right) \end{aligned} \quad (7)$$

where $i = 1, 2$ for the bars OA and OB, respectively, and $j = 1, 2$ for the two load systems. Under the action of the load Q_2 , the bar OB is in compression and is, hence, liable to buckle. However, it is assumed that the bar OB has a low slenderness ratio and will consequently fail by yielding of the material at the stress F before the critical buckling stress is reached.

Let g_{ij} denote the random variable $F - (R_{ij}/A_i)$; then

$$P_{ij} = P(g_{ij} < 0) \quad (8)$$

It is necessary to examine the random variable g_{ij} to be able to determine P_{ij} . In general

$$\begin{aligned} g_{ij} &= F - \frac{R_{ij}}{A_i} \\ &= F - \frac{Q_j}{C_{ij}(\theta)A_i} \end{aligned} \quad (9)$$

where $C_{ij}(\theta)$ is a function of θ . Now since θ is a constant, C_{ij} is a constant; also A_i is, in this distribution equation, a constant, and F and Q_j are normally distributed. Hence, g_{ij} is a normally distributed random variable. The mean and standard deviation of the distribution g_{ij} are given as mean

$$\bar{g}_{ij} = \bar{F} - \frac{\bar{Q}_j}{C_{ij}A_i} \quad (10)$$

standard deviation

$$\sigma_{g_{ij}} = \left(\sigma_F^2 + \frac{\sigma_{Q_j}^2}{C_{ij}^2 A_i^2} \right)^{1/2} \quad (11)$$

The constant C_{ij} is represented as follows:

$$\begin{aligned} C_{11} &= 2 \cos \theta, & \text{for bar OA under loading Q1} \\ C_{21} &= 2 \cos \theta, & \text{for bar OB under loading Q1} \\ C_{12} &= 2 \sin \theta, & \text{for bar OA under loading Q2} \\ C_{22} &= 2 \sin \theta, & \text{for bar OB under loading Q2} \end{aligned}$$

Now, the probability of failure

$$P_{ij} = \int_{-\infty}^0 \frac{1}{\sqrt{2\pi} \sigma_{g_{ij}}} e^{-\frac{(g_{ij} - \bar{g}_{ij})^2}{2\sigma_{g_{ij}}^2}} dg_{ij} \quad (12)$$

The probability of failure of the structure is

$$\left[P_{FQ1 \text{ or } Q2 \text{ or } (Q1 \text{ and } Q2)} \right] = P_{FQ1} + P_{FQ2} - P_{FQ1}P_{FQ2} \quad (13)$$

where

$$P_{FQ1} = 1 - (1 - P_{11})(1 - P_{21}) \quad (14)$$

$$P_{FQ2} = 1 - (1 - P_{12})(1 - P_{22}) \quad (15)$$

In the characteristics of the distribution g_{ij} (eqs. (10) and (11)), the quantities \bar{F} , σ_F , \bar{Q}_j , σ_{Q_j} , and C_{ij} are available from the given data. The only unknown quantity is A_i . Thus, the characteristics of g_{ij} are unknowns in A_i ; that is, they are unknown functions in the variable A_i . Hence, P_{ij} (eq. (12)) is also a function of the variable A_i . From equation (13), the probability of failure of the structure is an unknown function in the variables A_1 and A_2 (i.e., $P_F = P_F(A_1, A_2)$). The condition

$$P_F = P_F(A_1, A_2) \leq \alpha \quad (16)$$

is thus a constraint in the variables A_1 and A_2 . For the structural optimization problem, the next function is the weight W of the truss (given by eq. (6)), the constraint is represented by equation (16), and the design parameters are A_1 and A_2 . The synthesis method can then be employed to obtain the optimum set of design parameters A_1 and A_2 so that W is a minimum and P_F does not exceed α .

NUMERICAL EXAMPLES

The above sample problem is solved for the following two sets of data:

$$\bar{F} = 15,000 \text{ lb/in.}^2, \sigma_F = 500$$

$$\bar{Q}_1 = 30,000 \text{ lb}, \bar{Q}_2 = 30,000 \text{ lb}$$

$$\sigma_{Q1} = \sigma_{Q2} = 1,000$$

$$\theta = 45^\circ$$

$$(1) P_F = 0.01 \quad (2) P_F = 0.2$$

The graphical solutions of the problem are presented in figure 3. The optimum design parameters for the two cases are: (1) $A_1 = 1.615 \text{ in.}^2$, $A_2 = 1.615 \text{ in.}^2$ with the weight parameter $W/\rho l = 3.23 \text{ in.}^2$, and (2) $A_1 = 1.524 \text{ in.}^2$, $A_2 = 1.524 \text{ in.}^2$ with the weight parameter $W/\rho l = 3.048 \text{ in.}^2$.

On the other hand, if the idealistic version of failure ($P_F \doteq 0$) were adopted (i.e., if failure were prevented with certainty in designing the truss), then extreme values of loading and allowable stress would have to be considered. Let Q_E be the upper extreme value of loadings Q_1 and Q_2 such that the probability of the load exceeding Q_E is 0.987×10^{-9} ; then for the above data $Q_E = 36,000 \text{ lb}$. Let F_E represent the lower extreme value of the stress F such that the probability of the stress being less than F_E is

0.987×10^{-9} ; then for the above data $F_E = 12,000$ psi. For this case, then, the design parameters are $A_1 = 2.14$ in.², $A_2 = 2.14$ in.², and the weight parameter $W/\rho l = 4.28$ in.².

CONCLUDING REMARKS

The concept emphasized in this paper is the employment of probability of failure constraint in the mathematical programming problem to determine the set of design parameters which minimizes the weight of the structure. It is shown how the probability of failure constraint of a structure can be formulated in terms of the design parameters; then, knowing the optimizing weight function in terms of the design parameters defines the mathematical model of the structural optimization problem completely.

It is seen that for the same loading, a truss designed for a probability of failure of 0.2 is lighter than that designed for a probability of failure of 0.01. However, both these structures are much lighter than that designed to prevent failure absolutely, that is, designed for $P_F \div 0$. The probabilities of failure of 0.2 and 0.01 are realistic compared to the idealistic probability of failure of zero. If these probabilities are arrived at by a judicious compromise between the weight saving desired and the amount of risk that could be afforded, then it can be seen that considerable weight can be saved by designing the structure for realistic probabilities of failure.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Sept. 12, 1966
124-08-06-01-21

REFERENCES

1. Parzen, Emanuel: Modern Probability Theory and Its Applications. John Wiley and Sons, Inc., N. Y., 1960.
2. Ghista, Dhanjoo N.: Weight Optimization by Structural Synthesis. J. Sci. Eng. Res., vol. 10, 1966.

BIBLIOGRAPHY

1. Freudenthal, Alfred M.: Safety and the Probability of Structural Failure. Trans. Am. Soc. Civil Engrs., Paper No. 2843, pp. 1337-1397.
2. Hilton, Harry H., and Feigen, Morris: Minimum Weight Analysis Based on Structural Reliability. J. Aerospace Sci., vol. 27, no. 9, Sept. 1960, pp. 641-652.
3. Bouton, Innes: The Implementation of Reliability Concepts in Structural Design Criteria. Intern. Council Aeron. Sci., Aug. 24-28, 1964, Paper No. 64-571.
4. Switzky, Harold: Minimum Weight Design With Structural Reliability. J. Aircraft, vol. 2, no. 3, June 1965, pp. 228-232.
5. Switzky, Harold: Designing for Structural Reliability. J. Aircraft, vol. 2, no. 6, Nov.-Dec., 1965, pp. 509-516.

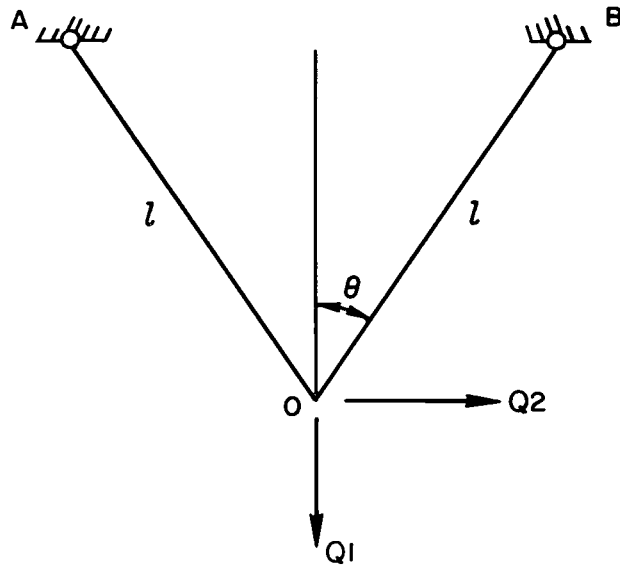


Figure 1.- Truss being considered for explanation of probability of failure constraint.

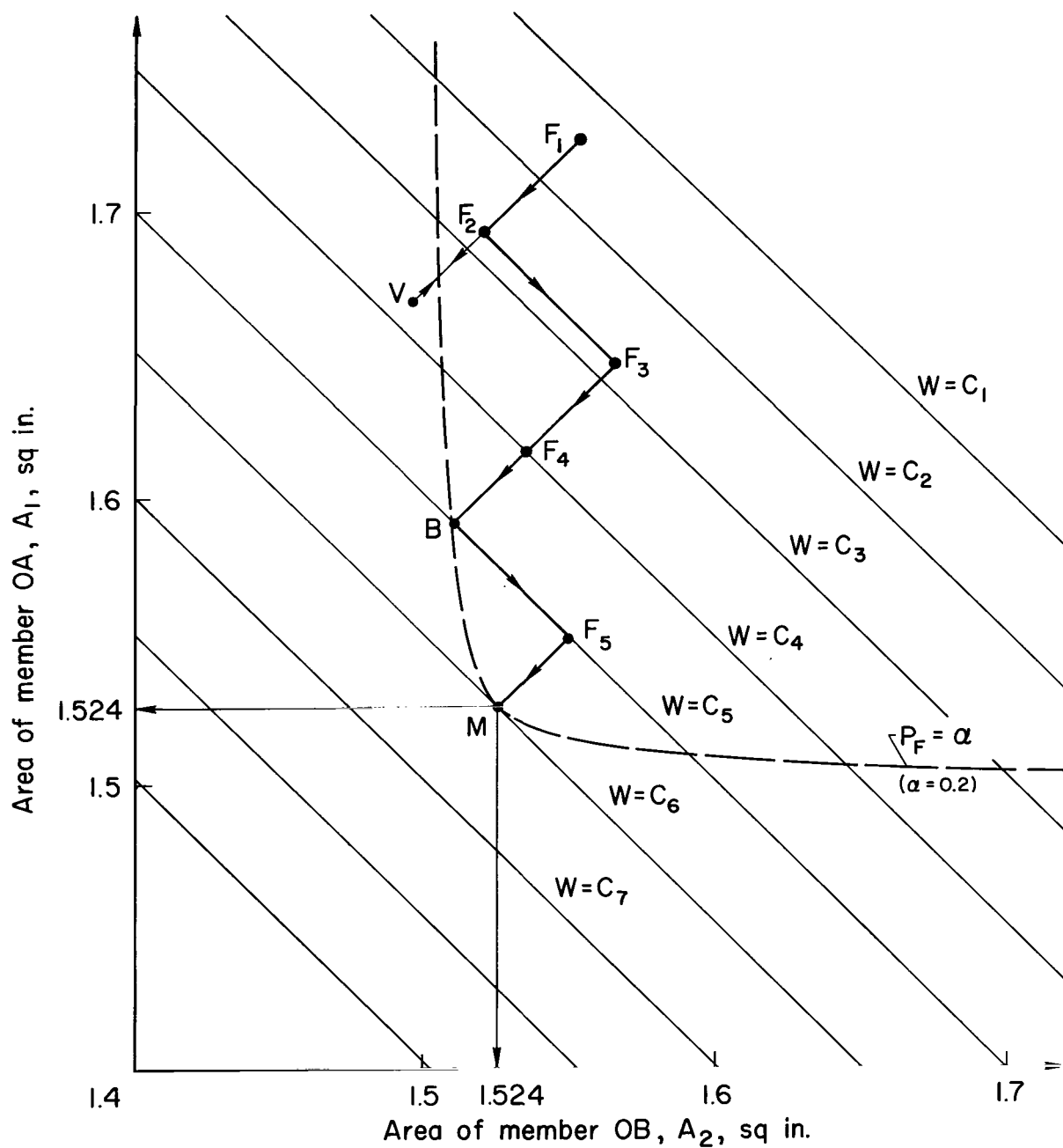


Figure 2.- Design parameter space for the truss of figure 1.

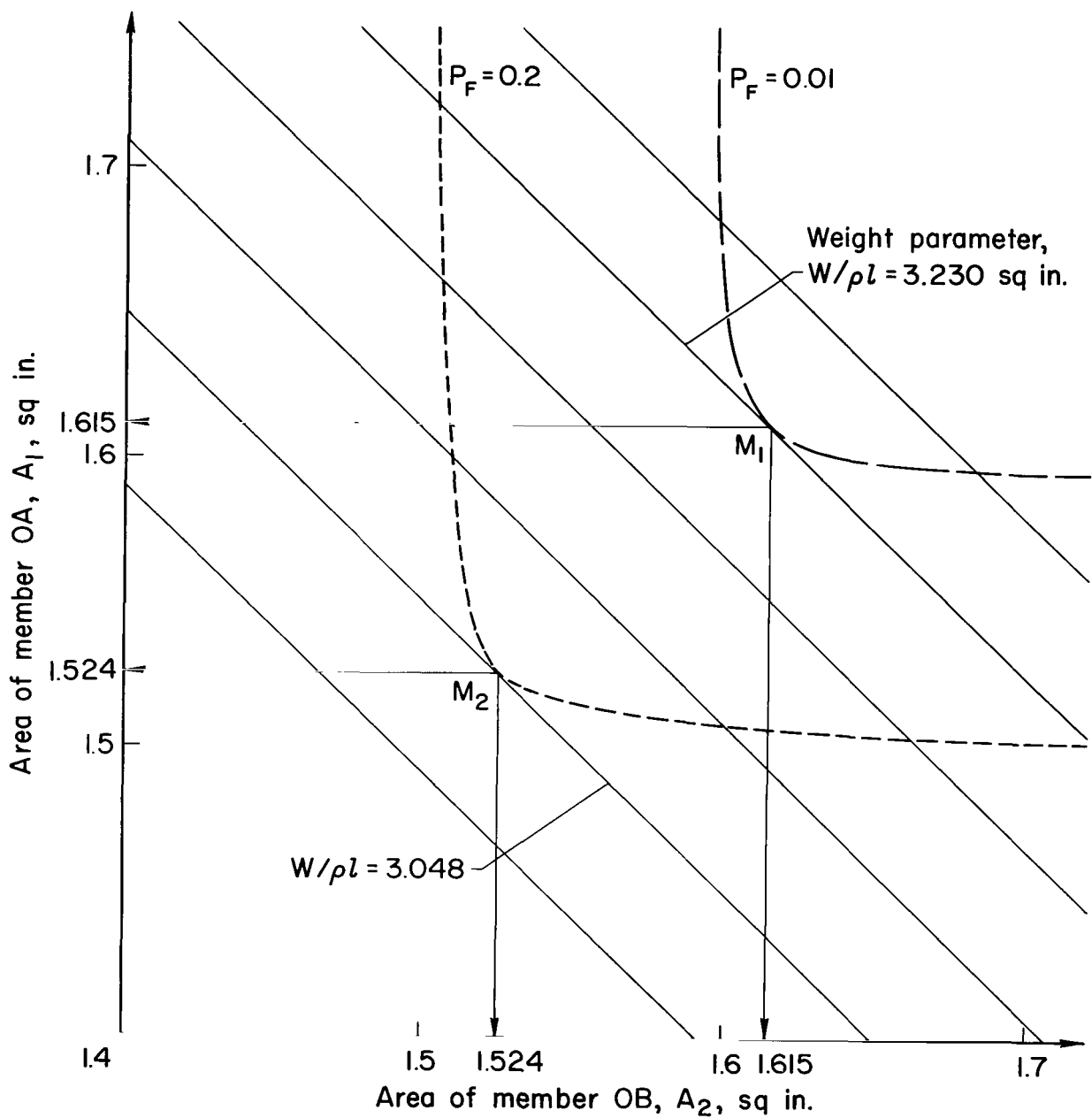


Figure 3.- Design parameter space showing the graphical solutions of the truss-optimization numerical examples.

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Technical information generated in connection with a NASA contract or grant and released under NASA auspices.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

TECHNICAL REPRINTS: Information derived from NASA activities and initially published in the form of journal articles.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities but not necessarily reporting the results of individual NASA-programmed scientific efforts. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C. 20546